

Revival-collapse phenomenon in the quadrature squeezing of the multiphoton Jaynes-Cummings model with the binomial states

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In this paper we study the interaction between two-level atom and quantized single-mode field, namely, Jaynes-Cummings model (JCM). The field and the atom are initially prepared in the binomial state and the excited atomic state, respectively. For this system we prove that the revival-collapse phenomenon exhibited in the atomic inversion of the standard JCM can be numerically (naturally) manifested in the evolution of the squeezing factor of the three-photon (standard) JCM provided that the initial photon-number distribution of the radiation has a smooth envelope.

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I. INTRODUCTION

Jaynes-Cummings model (JCM) [1] is one of the fundamental systems in the quantum optics. The simplest form of the JCM is the single quantized mode interacting with the two-level atom. Various phenomena have been realized for this system such as revival-collapse phenomenon (RCP) in the evolution of the atomic inversion [2], sub-Poissonian statistics and squeezing, e.g., [3]. Actually, the RCP represents the most important phenomena reported to this model since it manifests the granular nature of the initial field distribution [4] as well as the strong entanglement between the radiation field and the atom. The RCP has been observed via the one-atom mazer [5] and also using technique similar to that of the NMR refocusing [6]. It is worth mentioning that the RCP has been also remarked in the evolution of different quantities in the nonlinear optics such as the mean-photon number of

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the Kerr nonlinear coupler [7] and the photon-number distribution of the single-mode [8] and two-mode [9] squeezed coherent states with complex squeeze and displacement parameters. In the latter two cases the RCP occurs in the photon-number domain rather than in the interaction time domain.

There is an important class of nonclassical states, namely, binomial states. The binomial state (BS) is an intermediate state between the Fock state and the coherent state [10], i.e. it is linear combination of Fock states weighted by binomial distributions. The BS can exhibit many nonclassical effects, e.g., squeezing, sub-Poissonian statistics and negative values in the Wigner function [11]. Recently, the BSs have been proposed as reference field states in schemes measuring the canonical phase of quantum electromagnetic fields [12]. The BS can be generated by a classical current interacting with two quantized radiation fields [10, 13] as well as via quantum state engineering [14]. In the latter technique the resonant interaction of N' consecutive two-level atoms with the cavity initially prepared in its vacuum state is constructed and the desired cavity field state can be obtained from total state reduction by performing measurement on the atoms coming out of the cavity [15]. Quite recently, an efficient scheme for generating and detecting two-photon generalized binomial state in a single-mode high- Q cavity is described in [16]. Evolution of the BS with the JCM has been investigated for the single-photon JCM [17], the two-photon JCM [18] and the single-photon Kerr-nonlinear JCM [19]. The object of these studies is to investigate the construction of different phenomena when the field evolves gradually from the Fock state to the coherent state. It is worth referring that the evolution of the BS with the JCM can provide different behaviors than those with the coherent state, e.g., under certain conditions the evolution of the atomic inversion related to the BS exhibits a very steady beat phenomenon similar to that found in the classical physics [19]. The superposition of the BS (SBS) has been developed [20, 21] as:

$$|M, \eta\rangle_\epsilon = \sum_{n=0}^M C_n^M(\eta, \epsilon) |n\rangle, \quad (1)$$

where the coefficient $C_n^M(\eta, \epsilon)$ takes the form

$$C_n^M(\eta, \epsilon) = \lambda_\epsilon \sqrt{\frac{M!}{(M-n)!n!}} \eta^n (1 - |\eta|^2)^{[\frac{M-n}{2}]} [1 + (-1)^n \epsilon], \quad (2)$$

$$|\lambda_\epsilon|^{-2} = 1 + \epsilon^2 + 2(1 - 2|\eta|^2)^M \epsilon,$$

where M is a positive integer, $0 < |\eta| \leq 1$ and ϵ is a parameter taking one of the values 0, 1

and -1 corresponding to BS, even BS and odd BS, respectively. Throughout the investigation we consider η to be real. In the limiting cases $(\epsilon, \eta) \rightarrow (0, 1)$ and $(\eta, M) \rightarrow (0, \infty)$ such that $M\eta^2 = \alpha^2$ the state (1) reduces to the Fock state $|M\rangle$ and the superposition of the coherent state $|\alpha\rangle_\epsilon$ [22], respectively.

There is another type of the SBS, which is called the phased generalized binomial state [21]. This type of state is represented by the superposition of the even or odd binomial states. As an example we give the definition of the orthogonal-even binomial state as

$$|M, \eta\rangle_e = A \sum_{n=0}^{[M/4]} C_{4n}^M(\eta, 0) |4n\rangle, \quad (3)$$

where $C_{4n}^M(\eta, 0)$ can be obtained from (2) and A is the normalization constant having the form

$$A^2 = \frac{4}{1 + (1 - 2|\eta|^2)^M + 2\text{Re}(1 - |\eta|^2 + i|\eta|^2)^M}. \quad (4)$$

Using appropriate limit the state (3) tends to the orthogonal-even coherent state [23]. It is worth mentioning that the common property of the binomial states is that the probability of detecting m quanta when $m > M$ is zero. From the above information one can realize that the SBS is one of the most generalized states in quantum optics.

Recently, for the JCM it has been shown that there is a relationship between the atomic inversion and the quadrature squeezing [24, 25]. More illustratively, for particular type of initial states, e.g. l -photon coherent states, the squeezing factors can naturally provide complete information on the corresponding atomic inversion. Nevertheless, for the initial coherent state it has been numerically shown that the evolution of the quadrature squeezing of the three-photon JCM reflects the RCP involved in the atomic inversion of the standard, i.e. the single-photon, JCM. These relations have been obtained based on the fact that for the initial l -photon coherent state and coherent state the harmonic approximation is applicable. In this paper we show that these relations exist also for any arbitrary initial field states provided that their photon-number distributions have smooth envelopes. In doing so we study the evolution of the JCM with the SBS. For this system we obtain various interesting results. For instance, we show that the relations between the atomic inversion and the quadrature squeezing are sensitive to the interference in phase space. Additionally, the odd N th-order squeezing of the standard JCM with the even-orthogonal binomial state exhibits RCP as that of the corresponding atomic inversion. The motivation of these relations is

that the RCP exhibited in the evolution of the atomic inversion can be measured by the homodyne detectors [26]. This is supported by the recent developments in the cavity QED in which the homodyne detector technique has been applied to the single Rydberg atom and one-photon field for studying the field-phase evolution of the JCM [27].

We construct the paper in the following order: In section 2 we give the basic relations and equations related to the system under consideration. In sections 3 and 4 we investigate naturally and numerically the occurrence of the RCP in the higher-order squeezing. In section 5 we summarize the main results.

II. BASIC EQUATIONS AND RELATIONS

In this section we give the basic relations and equations, which will be used in the paper. Specifically, we develop the Hamiltonian of the system and its wavefunction as well as the definition of the quadrature squeezing. Also we shed light on the relation between the photon-number distribution and the atomic inversion.

The Hamiltonian controlling the interaction between the two-level atom and the k th-photon single-mode field in the rotating wave approximation is [28]:

$$\frac{\hat{H}}{\hbar} = \omega_0 \hat{a}^\dagger \hat{a} + \frac{1}{2} \omega_a \hat{\sigma}_z + \lambda (\hat{a}^k \hat{\sigma}_+ + \hat{a}^{\dagger k} \hat{\sigma}_-), \quad (5)$$

where $\hat{\sigma}_\pm$ and $\hat{\sigma}_z$ are the Pauli spin operators; \hat{a} (\hat{a}^\dagger) is the annihilation (creation) operator denoting the cavity mode, ω_0 and ω_a are the frequencies of the cavity mode and the atomic transition, respectively; λ is the atom-field coupling constant and k is the transition parameter.

We consider that the field and atom are initially prepared in the SBS (1) and the excited atomic state $|+\rangle$, respectively. Also we restrict the investigation to the exact resonance case. Under these conditions the dynamical state of the system can be expressed as

$$|\Psi(T)\rangle = \sum_{n=0}^M C_n^M(\eta, \epsilon) [\cos(T\nu_{n,k})|+, n\rangle - i \sin(T\nu_{n,k})|-, n+k\rangle], \quad (6)$$

where $T = \lambda t$, $\nu_{n,k} = \sqrt{\frac{(n+k)!}{n!}}$ and $|-\rangle$ denotes the ground atomic state. The atomic inversion associated with (6) is

$$\langle \sigma_z(T) \rangle = \sum_{n=0}^M P(n) \cos(2T\nu_{n,k}), \quad (7)$$

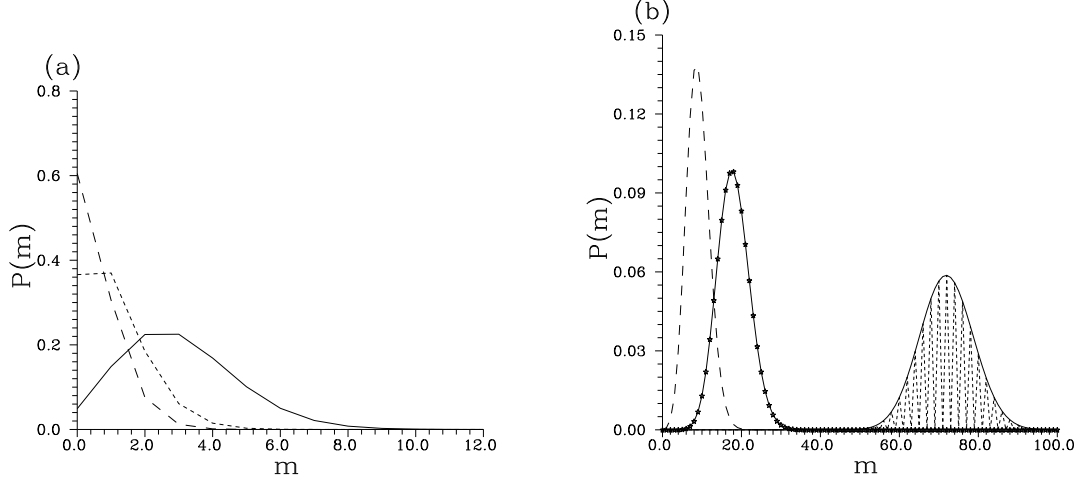


FIG. 1: The $P(m)$ against m for the BS. (a) $\eta = 0.1$ and $M = 50$ (long-dashed curve), 100 (short-dashed curve) and 370 (solid curve). (b) $(\eta, M) = (0.3, 100)$ (long-dashed curve), $(0.3, 200)$ (star-centered curve) and $(0.6, 200)$ (solid curve). The short-dashed curve in (b) is given for $(\epsilon, \eta, M) = (1, 0.6, 200)$.

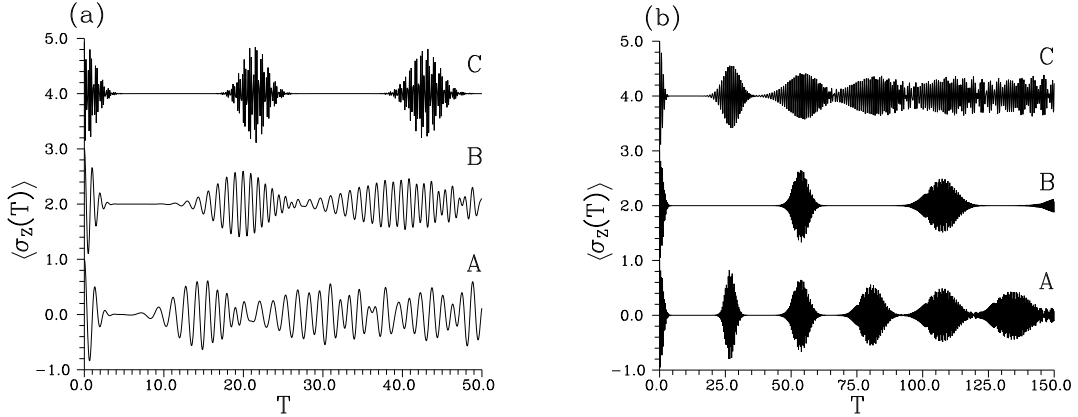


FIG. 2: The atomic inversion $\langle \sigma_z(T) \rangle$ against the scaled time T for $k = 1$ when the field is initially prepared in the SBS with different values of η , M and ϵ . (a) $(\eta, M, \epsilon) = (0.1, 370, 0)$ (curve A) and $(0.3, 100, 0)$ (curve B), whereas the curve C is given for the initial orthogonal-even binomial state with $(M, \eta) = (370, 0.7)$. (b) $M = 200$ and $(\eta, \epsilon) = (0.6, 1)$ (curve A), $(0.6, 0)$ (curve B) and $(0.3, 0)$ (curve C). The curves are shifted from the bottom by 0, 2, 4.

where $P(n) = |C_n^M(\eta, \epsilon)|^2$. To understand the relation between $P(m)$, $\langle \sigma_z(T) \rangle$ and quadrature squeezing we plot $P(m)$ and $\langle \sigma_z(T) \rangle$ in Figs. 1 and 2, respectively, for the given values of the parameters. Fig. 1(a) gives the development of the binomial state to the coherent

state. This is obvious from the solid curve in Fig. 1(a) as well as the curve A in the Fig. 2(a), which represents the RCP of the coherent state with $\alpha = \sqrt{M\eta^2} = \sqrt{3.7}$ (we have checked this fact). Generally, the comparison between the corresponding curves in Figs. 1 and 2 shows when $P(m)$ exhibits smooth envelope the $\langle\sigma_z(T)\rangle$ provides the RCP. Moreover, the interference in phase space manifests itself as two times revival patterns in the evolution of the $\langle\sigma_z(T)\rangle$ compared to those related to the BS (compare the curves A and B in Fig. 2(b)). Also from the dashed curve in Fig. 1(b) the maximum value of the $P(m)$ is close to $m \simeq \bar{n} = \langle\hat{a}^\dagger(0)\hat{a}(0)\rangle$. For the future purpose we have plotted curve C in Fig. 2(a) for the $\langle\sigma_z(T)\rangle$ of the initial orthogonal-even BS.

The different moments of the operators \hat{a}^\dagger and \hat{a} for the state (6) can be evaluated as

$$\begin{aligned} \langle\hat{a}^{\dagger s_2}(T)\hat{a}^{s_1}(T)\rangle &= \sum_{n=0}^{M-s_1} (C_{n+s_2}^M(\eta, \epsilon))^* C_{n+s_1}^M(\eta, \epsilon) \left[\cos(T\nu_{n+s_2,k}) \cos(T\nu_{n+s_1,k}) \frac{\sqrt{(n+s_1)!(n+s_2)!}}{n!} \right. \\ &\quad \left. + \sin(T\nu_{n+s_2,k}) \sin(T\nu_{n+s_1,k}) \frac{\sqrt{(n+k+s_1)!(n+k+s_2)!}}{(n+k)!} \right], \end{aligned} \quad (8)$$

where s_1 and s_2 are positive integers and $M < s_1$. Finally, the N th-order quadrature squeezing operators are defined by $\hat{X}_N = \frac{1}{2}(\hat{a}^N + \hat{a}^{\dagger N})$, $\hat{Y}_N = \frac{1}{2i}(\hat{a}^N - \hat{a}^{\dagger N})$, where N is a positive integer. The squeezing factors associated with the \hat{X}_N and \hat{Y}_N can be, respectively, expressed as [29]:

$$F_N(T) = \langle\hat{a}^{\dagger N}(T)\hat{a}^N(T)\rangle + \text{Re}\langle\hat{a}^{2N}(T)\rangle - 2(\text{Re}\langle\hat{a}^N(T)\rangle)^2, \quad (9)$$

$$S_N(T) = \langle\hat{a}^{\dagger N}(T)\hat{a}^N(T)\rangle - \text{Re}\langle\hat{a}^{2N}(T)\rangle - 2(\text{Im}\langle\hat{a}^N(T)\rangle)^2.$$

Now we are in a position to investigate the relation between the atomic inversion of the standard, i.e. $k = 1$, JCM denoting by $\langle\sigma_z(T)\rangle_{k=1}$ and the quadrature squeezing. This will be done in the following sections.

III. NATURAL APPROACH

Natural approach is based on the fact: the quantity $\langle\sigma_z(T)\rangle + \langle\hat{a}^\dagger(T)\hat{a}(T)\rangle$ is a constant of motion and hence $\langle\sigma_z(T)\rangle$ and $\langle\hat{a}^\dagger(T)\hat{a}(T)\rangle$ can carry information on each others [24]. Furthermore, this approach can be generalized to find a relation between $\langle\sigma_z(T)\rangle$ and

$\langle \hat{a}^{\dagger N}(T) \hat{a}^N(T) \rangle$, where N is a positive integer, as we show shortly. The discussion will be restricted to the case $k = 1$. Now the question is that for the JCM which type of the binomial states making the N th-order squeezing factors provide complete information on the corresponding atomic inversion? The answer to this question can be realized from (9). Precisely, when there is a type of binomial states satisfying simultaneously the conditions:

$$\langle \hat{a}^N(T) \rangle = 0, \quad \langle \hat{a}^{2N}(T) \rangle = 0. \quad (10)$$

In this case the squeezing factors reduce to $\langle \hat{a}^{\dagger N}(T) \hat{a}^N(T) \rangle$, which can be connected with the corresponding $\langle \sigma_z(T) \rangle$. As an example of these states is the orthogonal-even binomial states (3) provided that the squeezing order N is odd integer, i.e. $2N + 1$. For this case one can easily check that the conditions (10) are fulfilled and hence the squeezing factors reduce to

$$\begin{aligned} F_{2N+1}(T) &= \langle \hat{a}^{\dagger 2N+1}(T) \hat{a}^{2N+1}(T) \rangle = \langle \hat{a}^{\dagger 2N+1}(0) \hat{a}^{2N+1}(0) \rangle + (N + \tfrac{1}{2}) \langle \hat{a}^{\dagger 2N}(0) \hat{a}^{2N}(0) \rangle \\ &\quad - (N + \tfrac{1}{2}) A^2 \sum_{n=0}^{[M/4]} |C_{4n}^M|^2 \frac{(4n)!}{(4n-2N)!} \cos(2T\nu_{4n,1}). \end{aligned} \quad (11)$$

Using suitable limits for the summation in (11) and by means of the following relation

$$M! = (M - 2N)! M^{2N} \prod_{j=0}^{2N-1} (1 - \frac{j}{M}) \quad (12)$$

we arrive at

$$\begin{aligned} F_{2N+1}(T) &= \langle \hat{a}^{\dagger 2N+1}(0) \hat{a}^{2N+1}(0) \rangle + (N + \tfrac{1}{2}) \langle \hat{a}^{\dagger 2N}(0) \hat{a}^{2N}(0) \rangle \\ &\quad - (N + \tfrac{1}{2}) |\eta|^{4N} M^{2N} \left[\prod_{j=0}^{2N-1} (1 - \frac{j}{M}) \right] A^2 \sum_{n=0}^{[M/4-2N]} |C_{4n}^{M-2N}|^2 \cos(2T\nu_{4n+2N,1}). \end{aligned} \quad (13)$$

For finite (large) values of N (M) with $0 < \eta < 1$, i.e. the $P(m)$ has smooth envelope, we can use the substitutions $\nu_{4n+2N,1} \simeq \nu_{4n,1}$ and $|C_{4n}^{M-2N}|^2 \simeq |C_{4n}^M|^2$ and hence the expression (13) can be modified to give the rescaled squeezing factor $W_N(T) (= \langle \sigma_z(T) \rangle_{k=1})$ through the relation:

$$W_N(T) = \frac{2 \langle \hat{a}^{\dagger 2N+1}(0) \hat{a}^{2N+1}(0) \rangle + (2N + 1) \langle \hat{a}^{\dagger 2N}(0) \hat{a}^{2N}(0) \rangle - 2F_{2N+1}(T)}{(2N + 1) \langle \hat{a}^{\dagger 2N}(0) \hat{a}^{2N}(0) \rangle_b}, \quad (14)$$

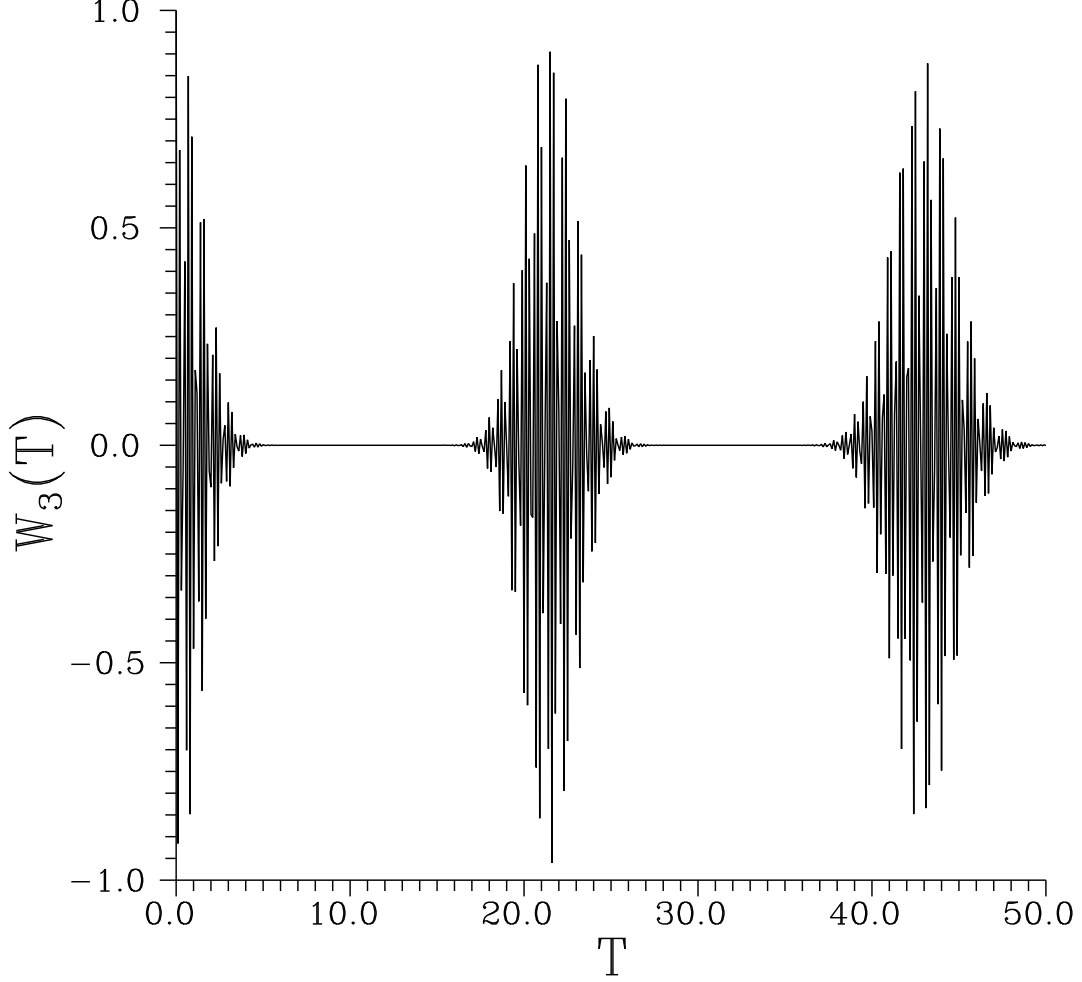


FIG. 3: The third-order rescaled squeezing factor given by (14) against the scaled time T , when the field is initially in the orthogonal-even BS with $(M, \eta) = (370, 0.7)$.

where the subscript b in the denominator means that the quantity $\langle \hat{a}^{\dagger 2N}(0) \hat{a}^{2N}(0) \rangle$ is related to the BS. Now we are in a position to check the validity of the (14). Thus we plot (14) in Fig. 3 for the third-order squeezing and the given values of the interaction parameters. We should stress that in Fig. 3 we have used the explicit form for F_{2N+1} given by (11). The comparison between the curve C in Fig. 2(a) and Fig. 3 demonstrates our conclusion: for particular type of binomial states the squeezing factor can provide complete information on the corresponding atomic inversion. The origin in this is that the expressions of the $\langle \sigma_z(T) \rangle$ and $\langle \hat{a}^{\dagger N}(T) \hat{a}^N(T) \rangle$ depend on the diagonal elements of the density matrix of the system under consideration.

IV. NUMERICAL APPROACH

In this section we study the possibility of obtaining information on the $\langle \sigma_z(T) \rangle_{k=1}$ from the squeezing factors of the k th-photon JCM, when the field is initially prepared in the SBS. Our object is to find the value of the transition parameter k ($k > 2$) for which one or both of the squeezing factors produce RCP as that involved in the $\langle \sigma_z(T) \rangle_{k=1}$ [24]. From (9) the RCP can likely occur in the $F_N(T)$ ($S_N(T)$) only when $\text{Re}\langle \hat{a}^N(T) \rangle = 0$ ($\text{Im}\langle \hat{a}^N(T) \rangle = 0$) since these quantities are squared, i.e. they destroy the RCP if it exists. According to this fact the occurrence of the RCP in F_N or in S_N depends on the values of the ϵ and N . Moreover, for $k > 2$ the quantity $\langle \hat{a}^{\dagger N}(T) \hat{a}^N(T) \rangle$ exhibits chaotic behavior and hence we can use $\langle \hat{a}^{\dagger N}(T) \hat{a}^N(T) \rangle \simeq \langle \hat{a}^{\dagger N}(0) \hat{a}^N(0) \rangle$. From this discussion we can conclude that if the squeezing factors exhibit RCP this will be related to the quantity $\text{Re}\langle \hat{a}^{2N}(T) \rangle$. Thus we treat this quantity in a greater details. From (8) and after minor algebra we arrive at

$$\begin{aligned} \langle \hat{a}^{2N}(T) \rangle &= \frac{|\eta|^{2N} M^N}{(1-|\eta|^2)^N} \sum_{n=0}^M |C_n^M(\eta, \epsilon)|^2 \left\{ \sqrt{\prod_{j=1}^{2N} \left(1 - \frac{(n+2N-j)}{M}\right)} \right\} \\ &\times \left[\cos(T\nu_{n+2N,k}) \cos(T\nu_{n,k}) + \sqrt{\prod_{j=0}^{2N-1} \frac{(1 + \frac{(k+2N-j)}{n})}{(1 + \frac{j}{n})}} \sin(T\nu_{n+2N,k}) \sin(T\nu_{n,k}) \right]. \end{aligned} \quad (15)$$

In (15) we have extended the upper limit of the summation from $M - 2N$ to M using the fact $l! = -\infty$ when $l < 0$ [30] because our goal is to compare this expression with that of the $\langle \sigma_z(T) \rangle_{k=1}$. Moreover, we assume that $M \gg 2N$, $0 < \eta < 1$ and \bar{n} is very large. Therefore, the quantity in the square root in the second line of (15) tends to unity. Additionally, for $\epsilon = 0$, the $P(n)$ exhibits smooth envelope and then the terms contributing effectively to the summation in (15) are those close to $n \simeq \bar{n} = M|\eta|^2$. In this case the quantity in the curly curves in (15) can be simplified as

$$\sqrt{\prod_{j=1}^{2N} \left(1 - \frac{(n+2N-j)}{M}\right)} = \sqrt{\prod_{j=1}^{2N} \left(1 - |\eta|^2 - \frac{(2N-j)}{M}\right)} \simeq (1 - |\eta|^2)^N, \quad (16)$$

where we have considered $\vartheta/M \rightarrow 0$ since ϑ is a finite c-number and $M \gg \vartheta$. On the other hand, when $\epsilon = 1$, say, the $P(n)$ exhibits oscillatory behavior with maximum value around $n \simeq \bar{n}$ (see the dashed curve in the Fig. 1(b)) and we arrive at

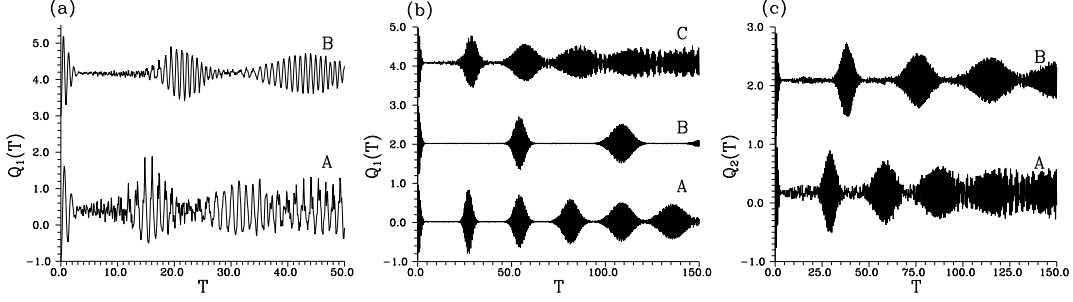


FIG. 4: The rescaled squeezing factors $Q_1(T)$ and $Q_2(T)$ as indicated against the scaled time T for different values of η, M and ϵ . (a) $(\eta, M, \epsilon) = (0.1, 370, 0)$ (curve A) and $(0.3, 100, 0)$ (curve B). (b) $M = 200$ and $(\eta, \epsilon) = (0.6, 1)$ (curve A), $(0.6, 0)$ (curve B) and $(0.3, 0)$ (curve C). (c) $(\eta, M, \epsilon) = (0.3, 200, 0)$ (curve A) and $(0.3, 370, 0)$ (curve B). The curves in (b) and (c) are shifted from the bottom by 0, 2, 4 and 0, 2, respectively, whereas in (a) are shifted by 0, 4.

$$\sqrt{\prod_{j=1}^{2N} \left(1 - \frac{(n + 2N - j)}{M}\right)} \simeq \left(1 - \frac{\bar{n}}{M}\right)^N = \left(1 - |\eta|^2 \frac{(1 - z^{M-1})}{1 + z^M}\right)^N = (1 - |\eta|^2)^N \left(\frac{1 + z^{M-1}}{1 + z^M}\right)^N, \quad (17)$$

where we have used the mean-photon number of the even binomial states as [20]:

$$\bar{n} = |\eta|^2 M \frac{(1 - z^{M-1})}{(1 - z^M)}, \quad z = 1 - 2|\eta|^2. \quad (18)$$

It is evident that $|z| < 1$ for $0 < \eta < 1$ and then $z^{M-1} \simeq 0$ where M is very large. Thus the result given by (16) is valid for all values of ϵ and hence the expression (15) reduces to

$$\langle \hat{a}^{2N}(T) \rangle \simeq |\eta|^{2N} M^N \sum_{n=0}^M |C_n^M(\eta, \epsilon)|^2 \cos[T(\nu_{n+2N,k} - \nu_{n,k})]. \quad (19)$$

Comparison between (7) (i.e. $\langle \sigma_z(T) \rangle_{k=1}$) and (19) shows that the two expressions can provide similar dynamical behavior only when the arguments of the cosines are comparable. This is regardless of the different scales resulting from the pre-factor $M|\eta|^{2N}$ in (19). The proportionality factor μ_N , say, which makes the dynamical behaviors in the two expressions

similar, can be evaluated from the following relation

$$\begin{aligned}\mu_N &= \frac{\nu_{n+2N,k} - \nu_{n,k}}{2\sqrt{n+1}}, \\ &= \frac{n^{\frac{k}{2}} \sqrt{\prod_{j=1}^k (1 + \frac{j}{n})} \left[\prod_{j=1}^{2N} (n+k+j) - \prod_{j=1}^{2N} (n+j) \right]}{2n^{2N+\frac{1}{2}} \sqrt{1+\frac{1}{n}} \sqrt{\prod_{j=1}^{2N} (1+\frac{j}{n})} \left[\sqrt{\prod_{j=1}^{2N} (1+\frac{k+j}{n})} + \sqrt{\prod_{j=1}^{2N} (1+\frac{j}{n})} \right]}.\end{aligned}\quad (20)$$

It is worth recalling that \bar{n} is very large, $P(n)$ exhibits smooth envelope, i.e. $n \simeq \bar{n}$, and the squeezing-order N is finite. Therefore, by applying the Taylor expansion for different square roots in (20) we obtain [25]:

$$\mu_N \simeq \frac{1}{4} [2Nk\bar{n}^{\frac{k-3}{2}} + \bar{n}^{\frac{k-5}{2}}(\dots) + \bar{n}^{\frac{k-7}{2}}(\dots) + \dots]. \quad (21)$$

From (21) it is evident that the RCP can occur in the squeezing factor only when $k = 3$ and hence $\mu_N = \frac{3N}{2}$. In this case we have neglected such type of terms $\bar{n}^{-1}, \bar{n}^{-2}, \dots$, where \bar{n} is very large. From the above investigation one can realize that the N th-order rescaled squeezing factor, which can give complete information on the $\langle \sigma_z(T) \rangle_{k=1}$, is

$$Q_N(T) = \frac{\langle \hat{n}(0) \rangle_b^N - V_N(T)}{\langle \hat{n}(0) \rangle_b^N}, \quad (22)$$

where

$$V_N(T) = \begin{cases} S_N(\frac{3T}{2N}) & \text{for } \epsilon = 0, \\ F_N(\frac{3T}{2N}) & \text{for } \epsilon = i, \\ S_N(\frac{3T}{2N}) = F_N(\frac{3T}{2N}) & \text{for } \epsilon = \pm 1, \quad N = 2m' + 1, \\ S_N(\frac{3T}{2N}) & \text{for } \epsilon = \pm 1, \quad N = 2m' \end{cases} \quad (23)$$

and m' is a positive integer. In the derivation of the formula (22) we have considered that the mean-photon numbers of the BS and the SBS are the same. This is correct for $0 < \eta < 1$ and large \bar{n} . It is worth mentioning that the formula (22) is valid for the initial superposition of the coherent states, too. Now we check the validity of (22) by plotting Figs. 4(a)–(c) for the given values of the interaction parameters. The comparison between the curves in Figs. 4(a)–(b) and the corresponding ones in the Figs. 2 leads to the following fact: when $0 < \eta < 1$ and \bar{n} is large, regardless of the values of ϵ , the $Q_1(T)$ copies well with the

$\langle \sigma_z(T) \rangle_{k=1}$. Nevertheless, when \bar{n} is relatively small (with $P(m)$ has a smooth envelope) the RCP can be established in $Q_1(T)$, but the overall behavior could be different from that of the $\langle \sigma_z(T) \rangle_{k=1}$. This result is obvious when we compare the curves A in Fig. 2(a) and Fig. 4(a), where one can observe $|Q_1(T)| > 1$. Fig. 4(c) is given for the higher-order squeezing. The comparison between the curve A in this figure and the curve C in Fig. 4(b) leads to that the normal squeezing can provide better information on the $\langle \sigma_z(T) \rangle_{k=1}$ than the amplitude-squared squeezing. Nevertheless, the information obtained from the higher-order squeezing can be improved by increasing the value of the \bar{n} (compare the curves A and B in Fig. 4(c)).

V. CONCLUSION

In this paper we have shown that for the JCM there is a relationship between the quadrature squeezing and the atomic inversion provided that the initial photon-number distribution exhibits smooth envelope. This fact has been proved using one of the most general quantum state, namely, the superposition of the binomial states. Precisely, we have shown that for particular types of the initial binomial states the N th-order squeezing factor can naturally give complete information on the corresponding atomic inversion. Also we have numerically shown that the N th-order squeezing factor of the three-photon JCM can provide complete information on the $\langle \sigma_z(T) \rangle_{k=1}$. These relations exist only when the $P(m)$ exhibits smooth envelope and \bar{n} is large. Moreover, as the squeezing order N increases the values of the \bar{n} have to be increased for getting better information from the $Q_N(T)$ on the atomic inversion. Finally, the results obtained in this paper are valid also when the field is initially prepared in the cat states.

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- [1] Jaynes E T and Cummings F W 1963 *Proc. IEEE* **51** 89.
- [2] Eberly J H, Narozhny N B and Sanchez-Mondragon J J 1980 *Phys. Rev. Lett.* **44** 1323; Narozhny N B, Sanchez-Mondragon J J and Eberly J H 1981 *Phys. Rev. A* **23** 236; Yoo H I, Sanchez-Mondragon J J and Eberly J H 1981 *J. Phys. A* **14** 1383; Yoo H I and Eberly J H 1981 *Phys. Rep.* **118** 239.
- [3] Shore B W and Knight P L 1993 *J. Mod. Opt.* **40** 1195.
- [4] Scully M O and Zubairy M S 1997 "Quantum Optics" (Cambridge University Press).
- [5] Rempe G, Walther H and Klein N 1987 *Phys. Rev. Lett.* **57** 353.
- [6] Meunier T, Gleyzes S, Maioli P, Auffeves A, Nogues G, Brune M, Raimond J M and Haroche S 2005 *Phys. Rev. Lett.* **94** 010401.
- [7] Korolková N and Peřina J 1997 *Opt. Commun.* **136** 135.
- [8] Dutta B, Mukunda N, Simon R and Subramaniam A 1993 *J. Opt. Soc. B* **10** 253.
- [9] Salvadoray M, Kumar M S and Simon R 1994 *Phys. Rev. A* **49** 4957.
- [10] Stoler D, Saleh B E A and Teich M C 1985 *Optica Acta* **32** 345.
- [11] Vidiella-Barranco A and Roversi J A 1994 *Phys. Rev. A* **50** 5233.
- [12] Pregnell K L and Pegg D T 2002 *Phys. Rev. Lett.* **89** 173601; Ibid 2003 *Phys. Rev. A* **67** 063814.
- [13] Lee C T 1985 *Phys. Rev. A* **31** 1213; Dattoli G, Gallardo J and Torre A 1987 *J. Opt. Soc. Am. B* **2** 185.
- [14] Faisal A A El-Orany, Abdalla M S, Obada A-S and Abd Al-Kader G M 2001 *In. J. Modern Phys. B* **15** 75.
- [15] Vogel K, Akulin V M and Schleich W P 1993 *Phys. Rev. Lett.* **71** 1816.
- [16] Lo Franco R, Compagno G, Messina A and Napoli A 2006 *Phys. Rev. A* **74** 045803.
- [17] Joshi A and Puri R R 1987 *J. Mod. Opt.* **34** 1421; Ibid 1989 **36** 557.
- [18] Joshi A and Puri R R 1989 *J. Mod. Opt.* **36** 215.
- [19] Du S-D and Gong C-D 1994 *Phys. Rev. A* **50** 779.
- [20] Abdalla M S, Mahran M H and Obada A-S F 1994 *J. Mod. Opt.* **41** 1889; Faisal A A El-Orany, Mahran M H, Obada A-S F and Abdalla M S 1999 *In. J. Theor. Phys.* **38** 1493.

- [21] Mahran M H, Sebawe M A, Obada A-S and Faisal A A El-Orany 1998 *Nonlinear Optics* **19** 189.
- [22] Schrödinger 1935 *Naturwissenschaften* **23** 807.
- [23] Lynch R 1994 *Phys. Rev. A* **49** 2800.
- [24] Faisal A A El-Orany 2004 *J. Phys. A: Math. Gen.* **37** 9023.
- [25] Faisal A A El-Orany 2005 *J. Phys. A: Math. Gen.* **38** 5557.
- [26] Mandel L and Wolf E 1995 "Optical Coherence and Quantum Optics" (Cambridge: University Press); Leonhardt U 1997 "Measuring the Quantum State of Light" (Cambridge: University Press).
- [27] Rauschenbeutel A, Nogues G, Osnaghi S, Bertet P, Brune M, Raimond J M and Haroche S 1999 *Phys. Rev. Lett.* **83** 5166; Bertet P, Auffeves A, Maioli P, Osnaghi S, Meunier T, Brune M, Raimond J M and Haroche S 2002 *Phys. Rev. Lett.* **89** 200402; Raimond J M, Brune M and Haroche S 2001 *Rev. Mod. Phys.* **73** 565.
- [28] Rosenhouse-Dantsker A 1992 *J. Mod. Opt.* **39** 1471.
- [29] Zhung Z-M, Xu L, Chui J-L and Li F-L 1990 *Phys. Lett. A* **150** 27.
- [30] Moya-Cessa H and Knight P L 1993 *Phys. Rev. A* **48** 2479.